

# KVS Mathematical Olympiad- 2015- (Stage I)

Time- 3 Hours

M. M -100

Note: All questions are compulsory. Each question carries 10 marks.

1. When the tens digit of a three digit number  $\overline{abc}$  is deleted, a two digit number  $\overline{ac}$  is formed. How many numbers  $\overline{abc}$  are there such that

$$\overline{abc} = 9\overline{ac} + 4c ?$$

2. Let  $p(x) = x^2 + bx + c$ , where  $b$  and  $c$  are integers. If  $p(x)$  is a factor of both  $x^4 + 6x^2 + 25$  and  $3x^4 + 4x^2 + 28x + 5$ , find the value of  $p(1)$ .
3. A square is inscribed in an equilateral triangle. Find the ratio of area of the square to that of the triangle.

4. (a) Prove that  $\frac{1}{2} - \frac{1}{3} + \frac{1}{4} - \frac{1}{5} + \dots + \frac{1}{98} - \frac{1}{99} + \frac{1}{100} > \frac{1}{5}$ .

(b) Find the largest prime factor of  $3^{12} + 2^{12} - 2 \cdot 6^6$ .

5. Surface area of a sphere A is 300% more than the surface area of another sphere B. If the volume of sphere B is  $p\%$  less than the volume of sphere A, find the value of 'p'.

6. ABC is an isosceles triangle in which  $AB = AC = 25$  cm and  $BC = 14$  cm. Find the difference of the circum-radius and in-radius of the triangle.

7. AB and BC are two equal chords of a circle of length  $2\sqrt{5}$  cm each. If radius of the circle is 5 cm, find the length of the chord AC.

8. Two dice are thrown simultaneously. Find the sum of the probability of "getting a prime number as a sum" and probability of "getting a doublet of prime numbers."

9. A person starts from a place P towards another place Q at a speed of 30 km/h. After every 12 minutes, he increases his speed by 5 km/h. If the distance between P and Q is 51 km., find the time taken by him to cover the whole distance.

10. Solve for 'x' ;

$$4\left(x - \frac{1}{x}\right)^2 + 8\left(x + \frac{1}{x}\right) = 29$$